NON-EUCLIDEAN GEOMETRY AND INFINITY

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A NEW PERSPECTIVE BY K. STRANG

Non-Euclidean Geometry

Hilbert Space

Hilbert space has infinite dimensions. In order to construct continuity from discrete or digitised space a 'curve' [1] is drawn through the centre of squares on a two dimensional plane, mapping points on a one-dimensional number line (see Figure 1). A function must be continuous if it is to represent a curve. When the number of squares on the plane is low, any mapped point may involve a discontinuity (a pseudo Hilbert Curve) but as the number of squares increase towards infinity a limit is reached and the pseudo curves converge to give a true space-filling curve.

A Platonic line or form of thinness itself can wander through an infinitely extending and richly dense space and hit every single point.

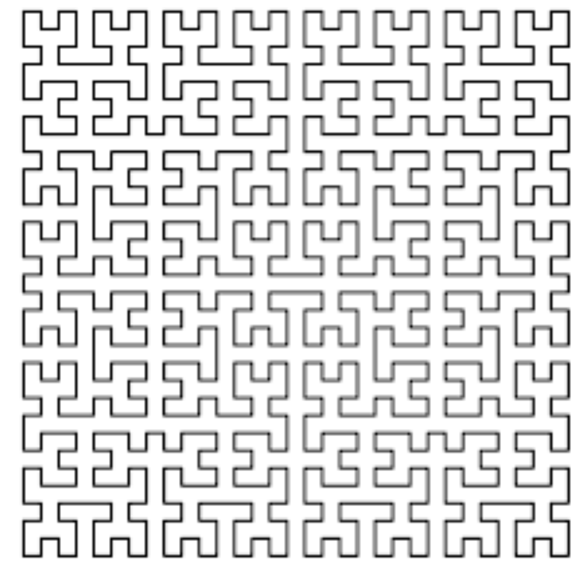

Figure 1

Fractals

Fractal geometry also looks to provide a solution to the infinity problem. The Mandelbrot set retains complexity no matter how close the computer zooms in. The paradox is that measurement of a line or an area stretches towards infinity the more detail is measured.

' "Indeed" Galileo proclaimed that "the great book of nature is written in mathematical language," adding that "its characters are triangles, circles and other geometrical figures, without which one wanders in vain through a dark labyrinth". Such Euclidian shapes are, however, quite useless in modelling either deterministic chaos, or irregular systems. These phenomena need geometries that are very far from triangles and circles. They require non-Euclidian structures – in particular, a new geometry called fractal geometry.' [Benoit Mandelbrot, T*he New Scientist* Guide to Chaos, Fractals - A *Geometry of Nature*, Penguin 1992, 123]

Fractal geometry captures essential features of natural phenomenon such as 'self-similarity', (have a look at pictures of the human brain) and 'bifurcation' (twigs, branches of trees). It also describes irregular systems such as population growth using the 'iteration' of a mathematical formula (ie the result of equation 1 is fed into equation 2 and so on – described as 'feedback'). But the interesting thing is that systems (produced by the repetition of an equation) [2] can converge towards an infinite cycle where it never repeats itself and leads to chaos. But within the chaos there are windows of stable periodic behaviour. See Figure 2.

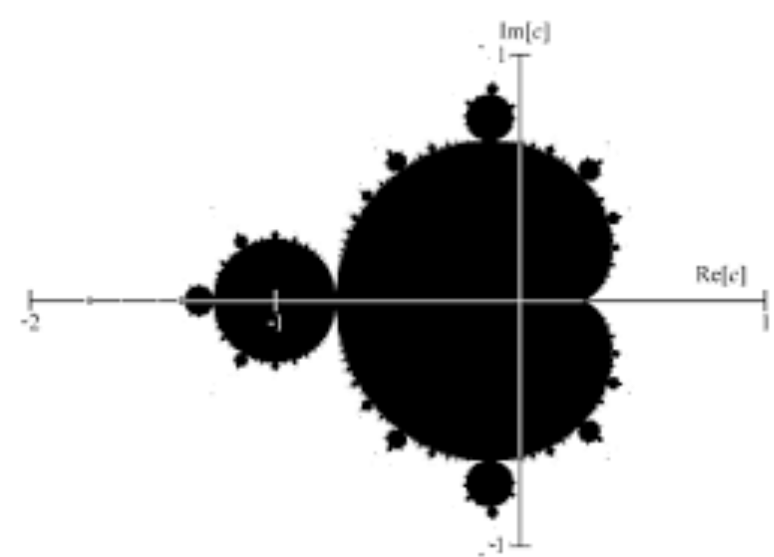

Figure 2

Hyperbolic

Hyperbolic geometry accepts the first four Euclidean postulates but not the fifth which states broadly that parallel lines cannot meet. This is used in GTR where space-time is not flat but curved. It is also used to reduce three dimensions to two dimensions and places a boundary on the infinite. See Figure 3 below.

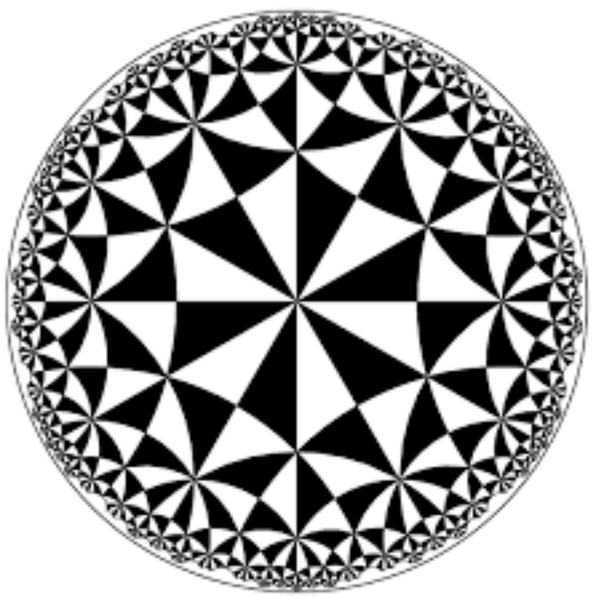

Figure 3

Infinity

Centuries ago, Zeno illustrated this problem. Who can forget the race between Achilles and the tortoise! Notwithstanding all the amazing scientific advances, it has not gone away. The 'measurement' problem in quantum physics has concentrated on the inability to include or integrate all aspects of the non-quantum measuring equipment into the closed quantum system. But an interesting development was the 'quantum zeno' effect where frequent measurements slowed or speeded up the progress of the wave function.

'The quantum Zeno effect (QZE) is stated to be a phenomenon in which the time evolution of a given quantum state slows down due to the frequent measurements of the system . . . This phenomenon seems to be opposed to the well- known measurement problem. Misra and Sudarshan were the first researchers who found this paradoxical behaviour through their theoretical investigations in 1971. On the other hand, the frequent measurements may have a reverse effect and accelerate the evolution under somewhat different conditions. This opposite effect is called the quantum anti-Zeno effect (QAZE) and occurs when the frequent measurements are not rapid enough . . .'

[*A new approach to study the Zeno effect for a macroscopic quantum system under frequent interactions with a harmonic environment* by Fatemeh Ghasemil & Afshin Shafiee, Nature Scientific Reports (2019) 9:15265 https://doi.org/10.1038/s41598-019-51729-1 1]

To my mind this seems reminiscent of the stroboscope effects: if a film is taken of a wagon wheel in motion in a clockwise direction and the frequency of each frame is faster than the angular frequency of the wheel, then the wheel looks as if it is rotating anti-clockwise or stationary. A stream of water droplets falling at regular intervals lit with a strobe light, is another example: when viewed under normal light, this is a normal water fountain but when viewed under a strobe light with its frequency tuned to the rate at which the droplets fall, the droplets appear to be suspended in mid-air. Adjusting the strobe frequency can make the droplets seemingly move up or down. So the question is whether there is a real ontological change in the process or just a perceptual change, or both?

Measurement at one time was the cornerstone of science and the touchstone of reality.

'**When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be.**[Lord Kelvin Popular Lectures and Addresses vol. 1 (1889) *Electrical Units of Measurement*, delivered 3 May 1883]

Statistical mechanics emerged as a separate discipline when it became impossible to treat all the molecules of a gas individually. This approach was supplemented by probability theory when Boltzman made the connection between the second law of thermodynamics and probable states.

Mathematics involves infinities: the number systems whether real, natural or imaginary are permeated by infinities. Georg Cantor (1845-1918) discovered different sizes of infinities using set theory: there is a larger infinity between zero and one than in the infinite range of natural numbers. [*Contribution to the Foundation of the Theory of Transfinite Numbers*, Dover Books 2018]

Space and time are measured in numbers. In any given space there are theoretically an infinite number of points and an infinite number of vibrations, so it is for example hardly surprising that at CERN they discover an endless stream of 'particles': gluons, muons, bosons, fermions, quarks etc.

Infinity is inherent in the notion of any quantity that can be measured numerically. Infinities can be dealt with mathematically as in calculus, infinite series, Fourier transformations, imposing Planck lengths etc. The question remains that if the phenomenal worlds of classical or quantum descriptions are permeated with infinities how can they be considered fundamental.

[1]Note: The 'curve' has angles (counter-intuitive) and can be thought of as a space-filling fractal.

[2] Note: The Mandlebrot set is more intricate as you zoom into it. It contains infinity in a finite shape – it is made up of countless examples of the Julia set – where real and imaginary numbers (e.g. 1 and √-1) are co-ordinates located on two axis, (real and imaginary) and, using the formula z-> z2+c, mapped onto another co-ordinate. Infinity is lurking in the finite and order and chaos are playfellows. It alters the notion of infinity as a series.