

RENORMALISATION



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A NEW PERSPECTIVE BY K. STRANG

Renormalisation

Quantum Electrodynamics QED was the first fully developed QFT attributed to Dirac when he synthesised QM and special relativity to explain the interaction of matter and light. His four linked equations gave rise to negative energy, anti-matter and anti-particles such as positrons. Even more extreme was the resulting infinities: when two electrons are as close as a Planck length they are impossible to analyse because electro-magnetic forces become infinitely large at small distances. Here is a summary:

‘Inside an atom of hydrogen, some ten volts is the gap between the electron and proton, which are separated on the average by only a tenth of a billionth of a metre. The resulting electric field is more than a thousand times greater than we can achieve in macroscopic technology, though this vast magnitude is restricted to atomic dimensions. Now we meet the enigma. In Dirac’s equation, the electron appears as a fundamental indivisible point of electric charge, and in the immediate vicinity of an electron with no physical extent, the field becomes infinitely strong. This could perhaps have been dismissed as a mathematical curiosity but for the physical implication. The electron interacts with its own field, like a snake biting its own tail, and gains energy known as “self-energy”. For the electron described by Dirac’s theory, the self-energy is infinite. Einstein’s theory of special relativity tells us that an amount of energy E represents an amount of mass = E/c^2 where c is the speed of light. The paradoxical result is that by interacting with its own electromagnetic field, an electron gains an infinite amount of inertia, or mass. As the mass of an electron has been measured, this infinite theoretical result is manifestly nonsense.’

and

Dirac’s equation assumed that an electron is no more than a piece of electric charge at a point in a spatial void. QED implied that as you voyage towards that elusive entity, you do so in the presence of an unseen swarm of ghostly spectators. These include the electromagnetic fields surrounding the electron, plus virtual electrons and positrons bubbling in and out of the vacuum. The “physical” electron is not the same as the ideal of Dirac’s equation. Instead, what experimentalists interpret as the

electron’s mass is the result of Dirac’s naked electron interacting with its own electromagnetic field, and also with the “vacuum polarisation” that fills the void. A “real” electron is a much more sophisticated thing than Dirac’s equation describes. The infinities in the QED calculations of an electron’s mass or electric charge hint at what you would find if you were able to measure with infinitely perfect resolution—revealing Dirac’s ideal point. [Frank Close *The Infinity Puzzle* OUP Oxford 2013]

In order to get rid of these unwelcome infinities, resort was had to some questionable mathematics, and processes such as Perturbation Theory and Renormalisation.

Perturbation theory is used when the solution to an equation is impossible or too difficult to solve: it allows an approximate expression which is simpler to solve and provides an approximate answer: it is an analytical method for determining approximate solutions of nonlinear equations for which exact solutions cannot be obtained. They are useful for demonstrating, predicting, and describing phenomena in vibrating systems that are caused by chaotic movement. Renormalisation is a method of obscuring or removing the unwanted infinities arising out of the equations, to give a result in finite quantities.

The Abstract from Freeman Dyson’s paper *Divergence of Perturbation Theory in Quantum Electro Dynamics* states:

‘An argument is presented which leads tentatively to the conclusion that all the power-series expansions currently in use in quantum electrodynamics are divergent after the renormalization of mass and charge. The divergence in no way restricts the accuracy of practical calculations that can be made with the theory, but raises important questions of principle concerning the nature of the physical concepts upon which the theory is built.’ [Physical Review Vol 85 No 4 Feb 15 1952]

Dirac expressed his disappointment with the outcome of his equation:

‘Hence most physicists are very satisfied with the situation. They say “ Quantum electrodynamics is a good theory, and we do not have to worry about it any more.” I must say I am very dissatisfied with the situation , because the so-called ‘good theory’ does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible math-

-ematics. Sensible mathematics involves neglecting a quantity when it turns out to be small, not neglecting it because it is infinitely great and you do not want it. [Dirac *Directions in Physics* (1978)z.Q *Electrodynamics*]

and Feynman on renormalisation:

‘ . . . no matter how clever the word, it is still what I would call a dippy process. Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that re-normalization is not mathematically legitimate.’ [*QED: The Strange Theory of Light and Matter*, 1990 Penguin p128]

Other physicists were adopting this questionable mathematics and telling the world that there was some counterpart in Nature, whereas what had been deduced was a formula for making predictions.

At first this seems very surprising that the theory has so much predictive power in describing the results of collisions in particle accelerators. But is it any more surprising than Newton magically turning his equation for gravity into a ‘force’ by applying an arbitrary number, elevated to the ‘gravitational constant’ for which there is no empirical foundation.

‘The various manipulations . . . may provide a sound method for generating well-defined functions but there remains a mystery about why they provide such good approximations to observable quantities measured in collider experiments. The renormalization procedure, in particular, seems to be flagrantly *ad hoc*. The process of regularizing integrals, redefining the coupling and removing the regulator in the standard renormalization procedure all seem to proceed in the absence of any physical argument underlying each step. . . . This appears to be bad news for the scientific realist, who wants to say that scientific predictions succeed because they are derived from theories that accurately represent the way the world is. On the other hand, my analysis of the perturbative approach does not fit neatly with the most popular forms of anti-realism. The constructive empiricist states their epistemic commitments with respect to models, taking them to be empirically adequate rather than

representationally faithful . . . it does not provide us with physical models at all, empirically adequate or not. *Prima facie* then, the sort of anti-realism motivated by the discussion so far would have to be a fairly radical form of instrumentalism, which takes the perturbative apparatus to be an algorithm for producing empirically successful predictions. Of course, instrumentalism has well known problems of its own . . .’ [James D Fraser, *The Real Problem with QFT* Brit. J. Phil. Sci. 71 (2020), 391–413, 409-410]

For a discussion on some more recent solutions to the problem see Charlie Wood *How Mathematical Hocus-Pocus Saved Particle Physics* Quanta Magazine, 17 September 2020.

As in Ptolemy’s epicycles, both edifices of current cosmology and particle physics are collapsing as an account of ontology.

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