

On dividing this expression by  $x^2 + p_1x + p_2$ , we obtain by the ordinary algebraic rules a remainder of the form  $M_{2s-1}x + N_{2s}$ , where  $M_{2s-1}$  and  $N_{2s}$  are functions of  $p_1$  and  $p_2$  whose weights are  $2s - 1$  and  $2s$  respectively, and which may accordingly be written in the forms

$$M_{2s-1} = b_{2s-1} + p_2 b_{2s-3} + \dots + p_2^{s-1} b_1,$$

$$N_{2s} = c_{2s} + p_2 c_{2s-2} + \dots + p_2^s,$$

where the  $b, c$  are of an order in  $p_1$  indicated by their suffixes. On writing down (by Professor Sylvester's Dialytic method) the result of eliminating  $p_2$  between these equations, it is at once apparent that this resultant is of the order  $s(2s - 1)$ . Thus the determination of a quadratic factor of an expression of degree  $2s$  is reduced to the solution of an equation of order  $s(2s - 1)$ . But this number is *one degree more odd* than the original number  $2s$ ; that is to say, if the number  $2s$  is  $2^r$  multiplied by an odd number, then  $s(2s - 1)$  is  $2^{r-1}$  multiplied by an odd number. Hence by a repetition of this process we shall ultimately arrive at an equation of odd order, which, as is well known, must have a real root. By then retracing our steps the existence of a quadratic factor of the original expression is demonstrated.

(3) *On the Space-Theory of Matter.* By W. K. CLIFFORD, B.A., *Trinity College.*

[*Abstract.*]

RIEMANN has shewn that as there are different kinds of lines and surfaces, so there are different kinds of space of three dimensions; and that we can only find out by experience to which of these kinds the space in which we live belongs. In particular, the axioms of plane geometry are true within the limits of experiment on the surface of a sheet of paper, and yet we know that the sheet is really covered with a number

of small ridges and furrows, upon which (the total curvature not being zero) these axioms are not true. Similarly, he says, although the axioms of solid geometry are true within the limits of experiment for finite portions of our space, yet we have no reason to conclude that they are true for very small portions; and if any help can be got thereby for the explanation of physical phenomena, we may have reason to conclude that they are not true for very small portions of space.

I wish here to indicate a manner in which these speculations may be applied to the investigation of physical phenomena. I hold in fact

(1) That small portions of space *are* in fact of a nature analogous to little hills on a surface which is on the average flat; namely, that the ordinary laws of geometry are not valid in them.

(2) That this property of being curved or distorted is continually being passed on from one portion of space to another after the manner of a wave.

(3) That this variation of the curvature of space is what really happens in that phenomenon which we call the *motion of matter*, whether ponderable or ethereal.

(4) That in the physical world nothing else takes place but this variation, subject (possibly) to the law of continuity.

I am endeavouring in a general way to explain the laws of double refraction on this hypothesis, but have not yet arrived at any results sufficiently decisive to be communicated.

*March 7, 1870.*

The President (Professor CAYLEY) in the Chair.

New Fellows elected:

W. G. ADAMS, M.A., *St John's College.*

A. T. CHAPMAN, M.A., *Emmanuel College.*

