THE ULTRA-VIOLET CATASTROPHE OR A STORM IN A TEACUP

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A NEW PERSPECTIVE BY K. STRANG

Historical Background

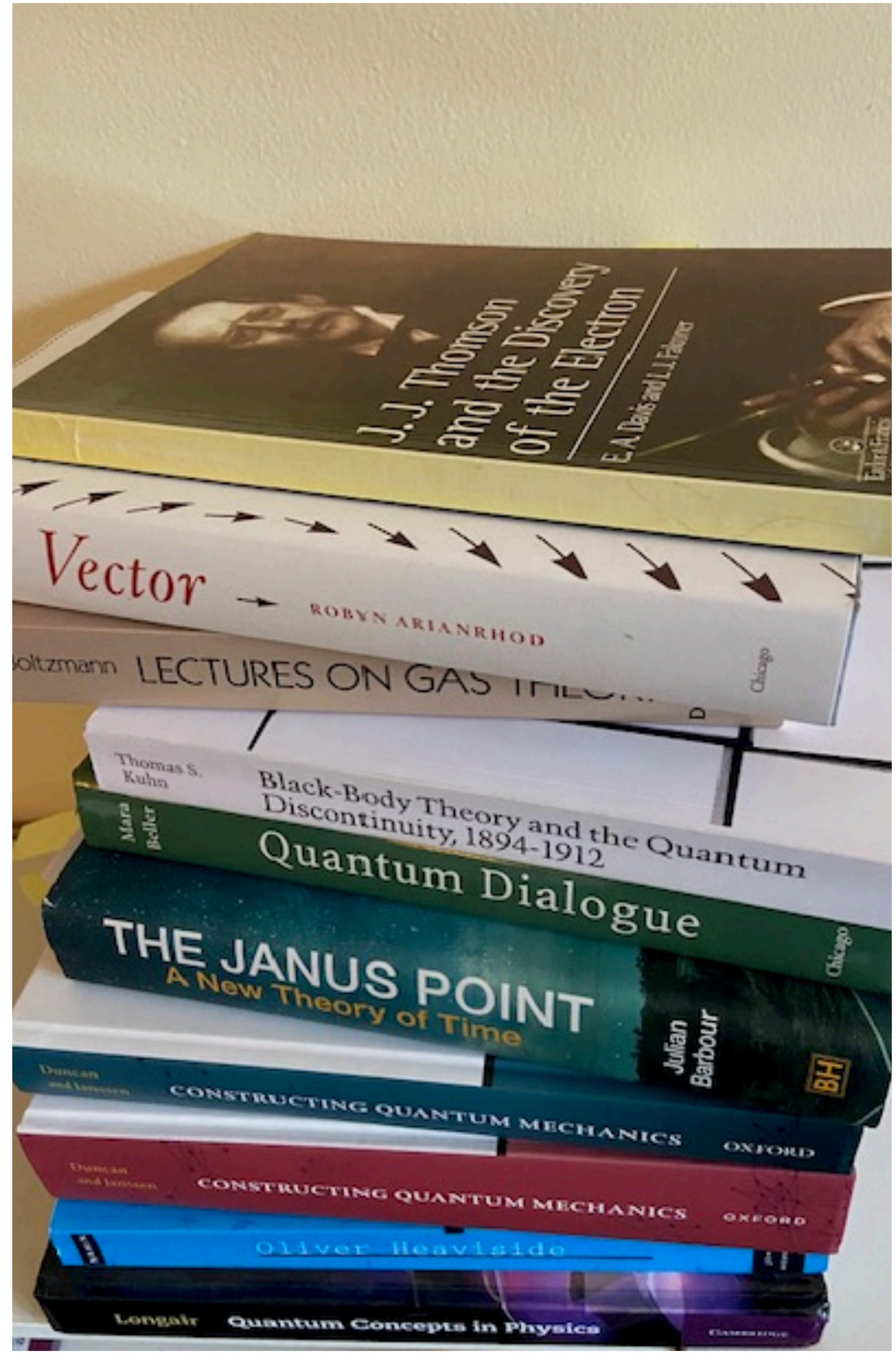
In the 19th century the three laws of thermodynamics (the study of heat energy) were discovered: (i) the conservation of energy; (ii) the irreversibility of physical processes or the increase in entropy and (iii) at temperatures of absolute zero molecular motion stops. Max Planck (1858-1947) dedicated a large part of his life looking for irreversible microscopic processes and he studied the waves produced by what he called oscillators or resonators (i.e., an object on a spring modelling a prototype atom).

Meanwhile other physicists, including Boltzman (1844-1906) and Maxwell (1831-1879) were inter alia applying statistical mechanics to ascertain the energy of a gas contained in a box. The basic idea behind statistical mechanics was to find a method of determining the kinetic energy of gas atoms or molecules. As it was considered impossible to work this out for any individual atom when dealing with many (i.e, Avogadro's Number 10^{23} per gram) bumping into each other and off the walls of their container, resort was had to finding statistical averages. This included the equipartition theorem which states that when a system (e.g., a container of gas) is in thermal equilibrium the kinetic heat energy will be shared equally across all the moving particles of that gas. So each particle will have an average energy of kT associated with each degree of freedom, (i.e. ways in which the molecule can move on the x, y and z axis) where k is the Boltzmann constant and T the temperature.

Concurrently, many scientists uncovered relationships between temperature and radiation and how radiation and matter interact. In 1859 G. R. Kirchhoff (1824-1887) published a paper *On the Emission and Absorption of Light* which concluded that energy emitted by a body in thermal equilibrium at any frequency is equal to the radiant energy absorbed at the same wavelength. An object that absorbs all of the light reflected on it is described as a 'black body'. If such an object has a temperature less than its surroundings it will continue to absorb energy (radiation) until a state of thermal equilibrium is reached; similarly if it is at a temperature higher than its surroundings it will emit energy (radiate heat) until equilibrium is achieved.

The Experiment

An experiment to demonstrate this theory, that black bodies absorb and emit radiation to reach thermal equilibrium was devised by James Jeans (1877-1946) and Lord Rayleigh (1842 -1919). Using a Jeans cube – a metal box with a small hole on one side, modelling as closely as possible an ideal black-body, they studied experimentally the radiation emitted and produced a graph, see the solid line in Figure 1 below. The results of their experiment confirmed the theory and supported two laws of physics: the Stephen-Boltzman law: as the temperature increases, the intensity (energy) increases; and Wein's



displacement law: as temperature increases the wavelength reduces: i.e. as more heat is applied to a piece of metal it will glow brighter and the colour will change from red to blue to white. As heat is removed the process will reverse until there is a state of equilibrium.

The Standard Account of the'Classical' theory

Jeans and Rayleigh wanted to find out if they could arrive at the practical result theoretically using a combination of the existing classical theories. Their calculations however produced a startling graph which looked like the dotted line in Figure 1 and implied that there would be an infinite amount of radiation emitted from the object at high frequencies, which would mean an infinite mass as $E = mc^2$. As this was impossible they concluded that classical physics could not solve this problem. Planck stepped in with his solution which will be analysed below.

The question is, was this a catastrophe or a storm in a teacup? Their methodology was wrong: they applied statistical mechanics and the equipartition theory which was designed to examine and average out the energy over a *finite* number of gas particles, to continuous wave phenomena. It was noted in Thomas Kuhn's book *Black Body Theory and the Quantum Discontinuity, 1894 - 1912* that many scientists questioned the use of the equipartition theory even for gas particles and claims that at the time, the equation could have been disproved but scientists were busy elsewhere.

'They could have refuted it at once, but apparently did not think it worth the effort. The Rayleigh-Jeans law and what came to be called the "ultra-violet catastrophe" did not yet pose a problem for more than two or three scientists.' [pp149-152]

Planck was one of those scientists and he initially tried to address the problem using the laws of thermodynamics but failed. In desperation he turned to statistics.

Planck's Solution

Planck solved the putative puzzle by quantising (or digitising) the waves into discrete elements or packages of energy, all multiples of a constant *h* and so made it possible to apply statistical mechanics coherently to wave phenomena. It should be noted that this statistical, mathematical device had originally been used by Boltzman who understood it was purely mathematical and did not lead to the ontological conclusion that discrete elements of energy existed.

Boltzman in his 1872 paper states:

'No molecule may have an intermediate or a greater kinetic energy. When two molecules collide, they can change their kinetic energies in many different ways. However, after the collision the kinetic energy of each molecule must always be a multiple of ε. I certainly do not need to remark that for the moment we are not concerned with a real physical problem. It would be difficult to imagine an apparatus that could regulate the collisions of two bodies in such a way that their kinetic energies after a collision are always multiples of ε. That is not the question here. In any case we are free to study the mathematical consequences of this assumption, which is nothing more than an artifice to help us calculate physical processes. For at the end we shall make ε infinitely small and pε infinitely large, so that the series of kinetic energies given in (18) will become a continuous one, and our mathematical fiction will reduce to the physical problem treated earlier. The kinetic energy of each molecule must always be a multiple of *ε* . . . we are not concerned here with a real physical problem . . . we are free to study the mathematical consequences of this assumption, which is nothing more than an artifice to help us calculate physical processes.'

Kuhn notes:

'To compute the entropy of an arbitrary distribution Planck must introduce combinatorials,[1] and for this purpose he follows Boltzman in subdividing the energy continuum into elements of finite size. It is at this point he introduces the further novelty that was soon to prove most consequential of all. For his purposes, unlike Boltzman's, the size of the energy elements e, e', e", etc must be fixed and proportional to the frequency.

" The distribution of the energy over each type of resonator must now be considered, first the distribution of the energy E over the N resonators with frequency *v*. If E is regarded as infinitely divisible, an infinite number of distributions is possible. We, however consider – and this is the essential point – E to be composed of a determinate number of equal finite parts and employ in their determination the natural constant *h* = 6.55 x 10^{-27} (erg x sec). **This constant multiplied by the frequency, v, of the resonator yields the energy element *ε* in ergs, and, dividing E by *ε*, we obtain the number P, of energy elements to be distributed over the N**

elements to be distributed over the N resonators.'" [ibid p 104-105; *Zur Theorie des Gesetzes* (Planck, 1900 pp239f; I,700f]

This strikes me as disingenuous as if E is infinitely divisible, E is infinite and dividing it by a finite quantity will also give an infinite number. This may be Planck using shorthand for E behaving like an infinite converging series along the lines described by Boltzman. If it is, then there is no doubt that the quantisation of energy is simply a mathematical device.

Also the derivation of the value of *h* as 6.55 x 10^{-27} (erg x sec) can be found in Planck's 1901 paper, *On the Law of the Energy Distribution in the Normal Spectrum,* Ann. Phys., 4, 553; translated by Kuyanov Yu. V. [kuyanov@mx.ihep.su] Planck uses measurements available from experiments to make the calculation. This is similar to Newton's gravitational constant which has no theoretical underpinning and was derived from experiment and really amounts to a way of translating the inverse square law into a force.[2] In the case of Planck's *h* measured as Joule second (SI Js^{1}), multiplied by frequency (*v*) measured in Herz (SI s^{-1}), allows the frequency to be converted to energy measured in SI units of Joules.

In defence of Planck, Kuhn notes that Planck was resistant to the idea of quantising radiation for some time and tried unsuccessfully to find a classical solution. Planck was generally not a fan of the atomic theory (the earlier corpuscular theory was discredited centuries ago by Leibniz) as it contradicted the second law of thermodynamics re irreversible processes: the collision between atoms can be reversed in time whereas a collision of waves cannot nor can a cold object spontaneously heat up.

'In conclusion I should like to call attention to a previously known fact. Consistently developed, the second law of the mechanical theory of heat is incompatible with the assumption of finite atoms. It can therefore be foreseen that the further development of the theory will lead to a battle between these two hypotheses in which one of them will perish . . . a variety of present signs seems to me to indicate that atomic theory, despite its great success, will ultimately have to be abandoned in favour of the assumption of continuous matter.' [Planck 1882 paper on *Vaporization, Melting and Sublimation*]

Kuhn also stresses this point**:**

'Fifteen years later Planck outlined the same position . . . Not mechanics but its atomistic formulation conflicted with the second law; . . . difficulties can presumably be eliminated by developing a mechanical world view of a continuum.' [ibid, Kuhn p23]

This is what Schrödinger attempted to do years later in his six papers on wave mechanics. Ultimately, as accepted historical accounts go, the discovery of *h* heralded a decisive break with classical physics (even although ironically, Newton held the view that light was a stream of particles), and Planck was the 'Father of quantum physics', a soubriquet he may not have wanted nor deserved.

Denouement

I never believed the conclusions drawn from this account, and could not understand why vector calculus had not been used to deal with the black-body problem, as it is the most appropriate mathematical tool for dealing with a continuous property. After all Maxwell's equations in the mid 19th century made use of this area of mathematics. Then I made a startling discovery: Maxwell's four very beautiful equations (see the additional material on the website) had been extrapolated from over 20 equations in his 1865 publication *A Dynamical Theory of the Electromagnetic Field,* by an Oliver Heaviside (1850-1925) who was one of the founders of vector calculus. The book *Vector Analysis* by the American J. Willard Gibbs (1839-1903) was published in 1901 around the time Planck had conjured up *h*.

Unfortunately, Oliver Heaviside is not mentioned in any account I have read of Maxwell's equations – not even in a footnote. You will see from the accompanying material that Maxwell's original equations are impenetrable and Faraday didn't care for them, preferring his pictorial account. Gibbs is mentioned by Kuhn, but only with reference to his prior book on *Statistical Mechanics*. Luckily there is an excellent biography of Heaviside, *Oliver Heaviside The Life, Work and Times of an Electrical Genius of the Victorian Age,* by Paul J. Nahin [The John Hopkins University Press, originally published by The Institute of Electrical and Electronics Engineers Press 1987]

'Maxwell wrote his 20 plus equations in cartesian component form in *A Dynamical Theory of the Electro Magnetic Field,* and in his Treatise he also mixed in quaternionic [3] concepts. Broadly, Heaviside whose hero was Maxwell, emphasised the fields as primary and jettisoned the vector potentials which he considered metaphysical, captured the symmetry of the fields and expressed the result in 4 equations in vector operator notation that most recognise today . . . they are an enormous compression of Maxwell's twenty equations.' [ibid, p100]

In 1881 Gibbs sent out review copies of the first part of *Elements of Vector Analysis* and in 1884, the concluding part, to *inter alia* Rayleigh, Kirchhoff, the two Thomsons (William and J. J.), Heaviside and Peter Tait (1831-1901). There was much resistance from the latter, who championed quaternions introduced by the Irish mathematician, William R. Hamilton (1805-1865) over vector analysis and that may account for its being overlooked when dealing with black-body radiation.

Heaviside nevertheless, persisted in using vector calculus:

'At the end of that year [Heaviside] wrote in an article in The Electrician "The relations between magnetic force and electric current," in which he mentioned Hamilton's quaternions and made plain his preference for more straightforward vector ideas on applications to physical theory'. [ibid, p196]

So the question is, if Planck had been *au fait* with vector calculus could he have solved the black-body problem without recourse to 'quantisation'. I believe the answer is 'yes' which would mean his discrete elements are an abstraction from an actual wave and have no ontological standing, and classical physics remains intact. It is worth noting that the discontinuity emphasised in quantum mechanics did not lay a glove on Maxwell:

'Maxwell's equations . . . have resisted the erosion and corrosion of progress. Special relativity had no effect on the equations (as magnetic phenomena are, in fact, relativistic effects, relativity is actually 'built in' to the equations!), and they were hardly perturbed at all by the arrival of quantum mechanics.' [ibid p85-86]

Why has no-one revisited black-body radiation and solved the problem using vector calculus? The truth is that statistical mechanics, a branch of classical physics, employed in gas theories based on atomic models of billiard balls or point particles, was shown to be seriously flawed, and this is what should have been abandoned, and a continuous wave theory developed. Instead the opposite happened: the standard model of particles was developed and radiation was quantised or construed as streams of particles. I believe this is one of the reasons why particle physics has ground to a halt.

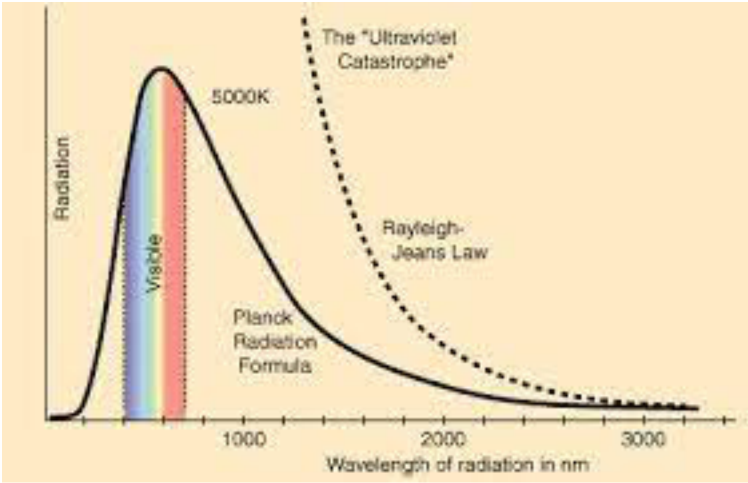


Figure 1

Notes

[1] Combinatorials is a method in mathematics of counting finite and discrete entities.

2 Newton had warned in Principia not to assume 'force' was physical: 'The words 'attraction', 'impulse' or any propensity to a centre, however I employ indifferently and interchangeably considering these forces not physically but merely mathematically. The reader should hence be aware lest he think that by words of this sort I anywhere define a species or mode of action, or a physical cause or reason.' This has been largely ignored by particle physicists.

[3] Quaternions, a 4-dimensional number that can describe transformations and rotations in 3-dimensional space. It was eventually replaced by vector calculus.