MAXWELL'S EQUATIONS ANNOTATED

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A NEW PERSPECTIVE BY K. STRANG

Maxwell's Equations Annotated

Gauss's Law

Gauss's law: $\operatorname{div} \mathbf{E} = \dfrac{\rho}{\varepsilon_0}$ $\qquad$ $\displaystyle\int_S \mathbf{E} \cdot \mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0} \int_V \rho \, \mathrm{d}V$

This covers charges in motion. Coulomb's law only covered stationary charges where the electric field of a stationary charge is radially directed, spherically symmetrical and falls off as the inverse square of the distance from the charge. It's the same format as Newton's gravity law. Gauss's law states that electric flux (ie the flow of electricity) over any closed surface S is equal to the total charge Q enclosed by the surface divided by a constant of proportionality (The permittivity of free space, denoted as ε_0 (epsilon nought), is a physical constant that measures the ability of electric fields to pass through a vacuum. Its value is approximately 8.854 x 10^{-12} farads per meter (F/m) and is essential in electromagnetism for calculating electric forces and fields.). The dimensions of the surface are irrelevant - all that matters is the amount of charge contained within.

Ampere's Law

No-monopole law: $\operatorname{div} \mathbf{B} = 0$ $\qquad$ $\displaystyle\int_S \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0$

This is equivalent to Gauss's law but covers magnetic flux. Unlike electric fields, magnetic fields form closed loops and have no monopoles (start or end points). The magnetic field is created by an unvarying or steady electric current.

Faraday's Law

Faraday's law: $\operatorname{curl} \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ $\qquad$ $\displaystyle\oint_C \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_S \mathbf{B} \cdot \mathrm{d}\mathbf{S}$

The law of electric induction and links magnetism and electricity while the above two laws deal with them separately. It states that the induced electro-magnetic field around a closed loop is equal to the rate of decrease of the magnetic flux over the open surface within the closed loop. Experiments
moving a magnet near a stationary current makes it move and vice versa. Faraday drew pictures of field lines to demonstrate this.

Ampere-Maxwell Law

Ampere–Maxwell law: $\operatorname{curl} \mathbf{B} = \mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \dfrac{\partial \mathbf{E}}{\partial t}$ $\qquad$ $\displaystyle\oint_C \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \int_S \left(\mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} \right) \cdot \mathrm{d}\mathbf{S}$

Ampere's law on its own only covers a steady unvarying current. Maxwell added a term to the RHS of the equation which makes no difference in steady situations but extends the law to cover systems which vary over time. It relies on the conservation of charge and the principle of continuity which links current and charge density.

Maxwell expressed these relationships in 20 equations in his paper *A Dynamical Theory of the Electro-Magnetic Field,* which were later compressed into 4 equations by Oliver Heaviside. The output of the four laws is to show that electromagnetism leads to electro-magnetic waves: i.e. light (radiation) is a self-propelled transverse electromagnetic wave.

Notes:
1. The equations can be expressed as integrals or partial differential equations because integration and differentiation are reversible.
2. B = Magnet Field; E = Electric Field, J = Current Density; Divergence and Curl are methods of putting a value on points or areas in the electric or magnetic field.
3. Symmetry is important in vector calculus as it provides a short cut to calculations and concentrates on spheres and cylinders because they can represent particles/charges or wires for currents.